SETON HALL UNIVERSITY
TWENTYFIRST ANNUAL
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MATHEMATICS COMPETITION

1. A car rental company charges $36 a day and 22¢ a mile for renting a car. June rented a car for 5 days and was charged $292.20. How many miles did she drive? (Answer: 510 miles)

2. The degree measure of each angle in a regular octagon is how much larger than the degree measure of each angle in a regular hexagon? (Answer: 15°)

3. Evaluate $a^{bc}+b^{ac}+c^{ab}$ if $a = 4$, $b = -1$, $c = 1/2$. (Answer: 17 1/2)

4. Jim drives a certain distance at an average rate of r miles per hour (where r is a positive real number). He then drives twice the original distance at an average rate of 2/3 r miles per hour. Find the average rate (in miles per hour) in terms of r for the total distance he drove. (Answer: 3/4)

5. Mixture A contains 45% alcohol and mixture B contains 75% alcohol. How many quarts of mixture A and mixture B must be combined to obtain 60 quarts of a mixture which contains 64% alcohol?
(Answer: 22 qts. of A; 38 qts. of B)

6. Simplify (where $t^2-2ct<0$): $\dfrac{(t-c)(c-t)}{\sqrt{2ct-t^2}}+\dfrac{c}{\sqrt{1-(t-c)^2/c^2}}+\sqrt{2ct-t^2}$. (Answer: 2 sqrt(2ct-t^2))

7. Let n be a positive integer and define $F(n)$ to be the sum of the $2n$ smallest positive integral multiples of n. For example, $F(3)=1\cdot 3+2\cdot 3+3\cdot 3+4\cdot 3+5\cdot 3+6\cdot 3=21\cdot 3=63$. Find the largest prime number p for which $F(p)<15000$. (Answer: 19)

8. The real values of x for which $\dfrac{8x^3+48x^2+117x+108}{x^3+9x^2+27x+27}\leq 4$ lie on an interval. Find the length of this interval. (Answer: 3)

9. One code (a "letter code") consists of 3 letters; the letters can be chosen from A to Z, but the third cannot be an O or an I; a letter can appear at most twice and repeated letters must be in adjacent positions. A

second code (a "digit code") consists of 4 digits; the digits can be chosen from 0 to 9, but the first cannot be a 0 or a 1; a digit can appear at most twice and repeated digits must be in adjacent positions; and two pairs of double digits are allowed. By how much does the possible number of "letter" codes exceed the possible number of "digit" codes? (Answer: 9783)

10. An ellipse lies on a coordinate plane and passes through the origin and the points (0,16) and (-4,0). If the major axis of the ellipse is parallel to the y-axis and is 34 feet long, find the length of the minor axis of the ellipse. (Answer: 68/15)

11. The real value of x for which $2^{4x+3} \cdot 3^{-3x+2} = 4^{2x+2} \cdot 5^{-x+1}$ can be written in the form $x = \dfrac{\log R}{\log T}$, where R and T are rational numbers. Find R and T (in simplest rational form). (Answer: R=10/9; T=5/27)

12. The Lodi Loops are to play the Bogata Bouncers in a best of 5 series. The first two games are to be played in Lodi, the next two in Bogata (if a fourth is needed), and the fifth in Lodi (if needed). The probability that Lodi will win at home is 2/3 (and that Bogata will win at Lodi is 1/3); the probability that Lodi will win at Bogata is 1/2 if they have won at least one previous game and is 3/8 if they have won no previous games. Find the probability that Bogata wins the series in fewer than 5 games. (Answer: 29/144)

13. A toy store manager purchased a total of 43 items including blocks (at \$8.50 each), dolls (at \$12.20 each), trucks (at \$10.40 each), and puzzles (at \$6.40 each), for a total of \$410.00. The amount spent for the trucks and puzzles exceeded the amount spent for the blocks and dolls by \$30. The number of blocks and trucks purchased was three more than the number of dolls and puzzles purchased. How much was spent on the trucks? (Answer: \$156)

14. Find the sum of the squares of the three values of x for which $\left(x^3 - x - 5\right)^3 - 3\left(x^3 - x - 5\right)^2 + 3\left(x^3 - x - 5\right) - 1 = 0.$ (Answer: 2)

15. Triangle ABC has sides AB, AC, BC of lengths 24 feet, 40 feet, 56 feet respectively. Line segment AD is drawn from A to point D on side BC, and AD bisects angle BAC. Find the length of line segment AD. (Answer: 15 ft.)

16. Let A and N be positive real integers. Bob earns $2A$ dollars the first week, $A/4$ more than that the second week, and so forth, each successive week earning $A/4$ more than the previous week. Ron earns A dollars the first week, $A/2$ more than that the second week, and so forth, each successive week earning $A/2$ more than the previous week. After both have worked for N weeks, Ron has earned \$252 more than Bob. The sum of Bob's earnings in the third week and Ron's earnings in the second from last week was \$100. How much did Bob earn in N weeks? (Answer:\$756)